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A Network of Integrate and Fire Neurons for Community Detection in Complex Networks

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Abstract

One salient feature of complex networks is the presence of communities, or groups of densely connected nodes. Community detection can not only help to understand the topological structure of complex networks, but also provide new techniques for real applications, such as data mining. In this paper, we propose a new model for community detection by using the synchronization and desynchronization property emerged from a network of Integrate and Fire neurons. This model has been applied to artificial and real-world networks and good results are obtained.
Keywords: Community Detection, Synchronization, Integrate & Fire Neurons.

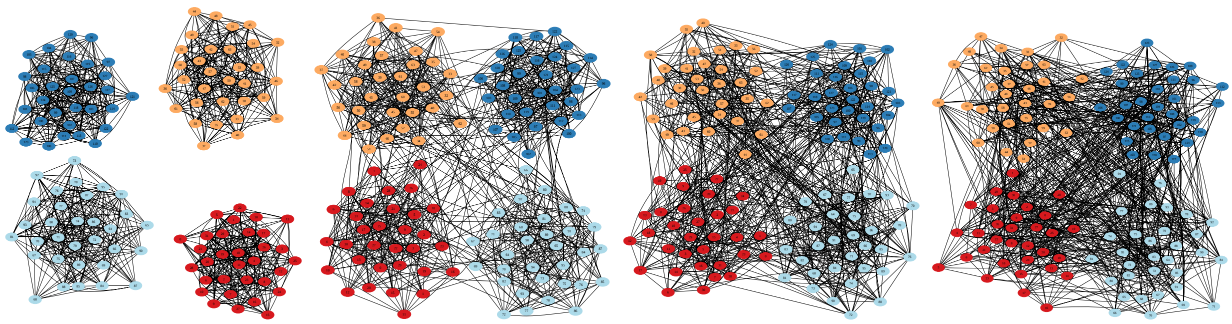
1. INTRODUCTION

A notable characteristic observed in diverse complex networks is the presence of local modular structures called *communities* [1, 2]. Such communities can be defined as groups of densely connected vertices, whereas connections between vertices pertaining to different groups (communities) are sparse [3]. These communities can represent patterns of interaction among vertices and its identification is important for understanding the growth and the formation mechanisms of the network [4]. Another important factor about the structure of communities lies in the similarity of vertices that compose them. Thus, by means of the identification and study of communities it is possible to obtain pertinent information about the domain of the network. For example, by scrutinizing the structure of links between pages of the *World Wide Web* is possible to observe that ones describing related subjects are more densely connected among than that with the remaining pages of the WWW [5]. This property is also shared by real networks in other domains, such as biological networks [6], metabolic networks [7], air transportation routes [8], among others.

Detecting communities in a network is not a computationally trivial process. For example, a simple version of this problem, the graph bi-partitioning problem, which consists of dividing a graph in two parts of the same size in such a manner that the number of links between these two parts is minimum, is a NP-Complete problem [2]. To make things worse, in real networks we do not know the number and the size of the communities and also how they are organized. Moreover, each community itself can be formed by other sub-communities in a hierarchical manner [2, 9]. Due to its importance in real applications and its high computational complexity, many researchers have proposed techniques to perform community detection automatically in complex networks [2, 3, 10–14].

In [2], a comparative study of various techniques was presented using the methodology described above. In this study it was observed that, for networks where the communities are well defined, it is quite easy to detect them and most of algorithms show a good precision. However, as the proportion of intercommunity links approach the proportion of intra-community links, the precision of those algorithms are reduced. In this situation, where the communities are not well defined, just a few techniques are able to detect communities with a precision higher than 80%. Although, as a negative quality, those techniques have a high computational complexity and can seldom be applied to large networks

Taking the tradeoff between precision and efficiency into account, in this work a new technique for community detection based on the *Oscillatory Correlation* theory is proposed. Accordingly to von der Malsbursg [15] the investigation of the brain functions and the perceptual organization indicates a temporal correlation mechanism as a representation framework in the brain. The temporal correlation theory defines that an object is represented by the temporal correlation of the firing activity of neurons coding different features of this object, while activity of neurons coding features of different objects are not correlated in time. A natural way of encoding the temporal correlation theory is through the use of oscillators [16, 17]. Thus, each object is represented by an assembly of oscillators with synchronous activity whereas



(a) (b) (c) (d)

Figure 1 – Examples of random clustered networks with $N = 128$, $M = 4$, and $\langle k \rangle = 16$. (a) $z_{out}/\langle k \rangle = 0.0$; (b) $z_{out}/\langle k \rangle = 0.1$; (c) $z_{out}/\langle k \rangle = 0.3$; (d) $z_{out}/\langle k \rangle = 0.5$.

distinct objects are represented by desynchronized groups of oscillators. This special form of temporal correlation is called *Oscillatory Correlation* [17–19] and it is the base of our model. Here, each oscillator corresponds to a vertex in the network in such a way that densely connected group of neurons, representing communities, have their firing activity synchronized, while the firing activities of distinct communities are not correlated in time owing to the absence or reduce number of links between them.

Another motivation for developing high precision and efficient techniques for community detection is its capability to reveal topological structures of the network. Based on this fact, those techniques are also important in machine learning, such as data clustering [20–22]. Generally speaking, the community structure reveals similarities by means of connections between vertices. These similarities can expose clustering in the data, and likewise reveal classes in classification problems. Moreover, by representing data in a network, classes or clusters of nontrivial shapes can be produced. Consequently, the development of new techniques for community detection can lead to the development of new algorithms for machine learning.

This work is organized as follows. Section 2 describes our model. The simulations are presented in Section 3. Finally, Section 4 draws some conclusions.

2. MODEL DESCRIPTION

In this model, each vertex of the network is represented by an Integrate and Fire (I&F) neuron [23] coupled by two connection types: excitatory connections and inhibitory connections. The first one defines a cooperative mechanism responsible for synchronizing group of neurons densely connected (a community). In contrast, the inhibitory connections, defined by a global inhibitor, have the purpose of breaking the synchrony between groups, which means, segregating the communities.

Each vertex is modeled by an I&F neuron defined by the following equation:

$$\frac{dv_i}{dt} = -v_i + I_i + E_i(t) - Y_i(t) \tag{1}$$

where v_i represents to the potential of the neuron i, I_i defines the external stimulation, $E_i(t)$ defines the excitatory coupling term, and $Y_i(t)$ the inhibitory signal from the global inhibitor. The neuron i fires when its potential $v_i \geq \theta_v$, where θ_v represents a firing threshold.

The excitatory coupling term $E_i(t)$ is defined by:

$$E_i(t) = \sum_{j \in \Delta_i} \omega_{ij} \delta(t - t_j) \tag{2}$$

where δ is the delta Dirac function, t_j represents the firing time of neuron j, Δ_i is the cooperative neighborhood of neuron i defined by network connections. ω_{ij} represents the excitatory coupling strength between neuron i and j and is defined as follows:

$$\omega_{ij} = \frac{c_E}{|\Delta_i|} \tag{3}$$

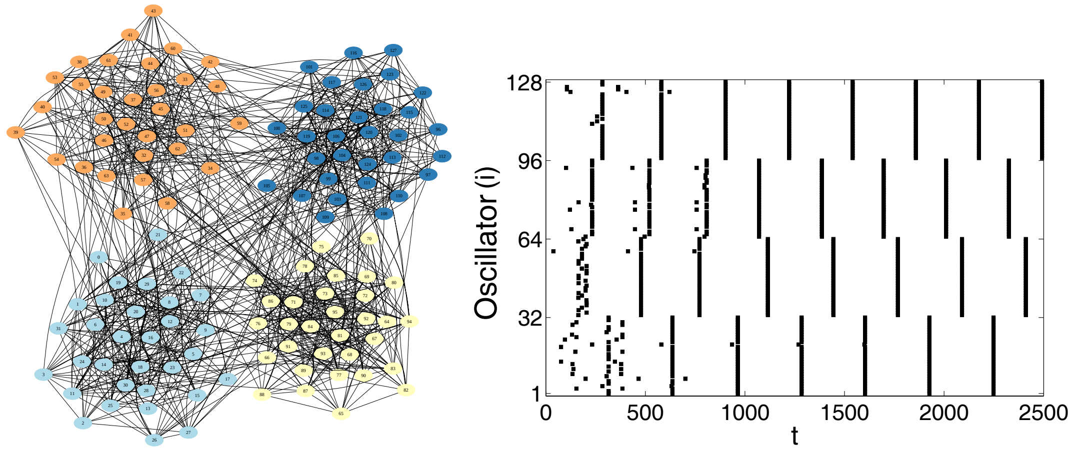
where $c_E \in [0, 1]$ is a parameter and $|\Delta_i|$ represents the degree of vertex i in the network. If c_E is set to a high value, neurons can become synchronized easily. On the other hand, if a very low value is used, the synchrony is hardly achieved.

The inhibitory coupling term (global inhibitor) is defined by:

$$Y_i(t) = \frac{c_Y}{N} \sum_{j=1}^{N} \delta(t - t_j) \tag{4}$$

where $c_Y \in [0, 1]$ is a parameter that defines the inhibition strength and N represents the number of neurons (vertices) in the network. If parameter c_Y assumes a high value, the inhibitory signal becomes stronger and more communities with small size are detected. On the contrary, when a low value is used, the synchrony among group of neurons becomes easier resulting in a detection of fewer communities composed of larger number of vertices.

Generally speaking, the model's dynamics can be described as follows. Owning to the excitatory connections, modeled by Equation (2), groups of neurons (vertices) densely connected, communities, have their firing activity synchronized. On the other hand, because of the presence of the global inhibitor (Equation (4)) associated with a smaller probability of intercommunity links, the firing activity of neurons coding vertices from different communities are not synchronized. For this reason, the proposed model is able



(a) (b)

Figure 2 – Illustration of the community detection process using the our oscillatory correlation model. In this simulation, $N = 128$, $M = 4$, $\langle k \rangle = 16$, $z_{out}/\langle k \rangle = 0.2$, and $c = 0.1$. (a) Input random clustered network. (b) Time series of oscillators (black dots represent neuron spikes).

to detect communities in networks in such a way that each community has its own distinct temporal activity. Moreover, an important characteristic of this approach is its simple dynamics and its fast synchronization, which results in a fast algorithm.

Next section presents the computer simulations with the proposed model using artificial and real networks.

3. COMPUTER SIMULATIONS

This section presents a set of simulations to test the capacity of our model as a computational tool for community detection. In all these simulations, except the last, the parameters $c_E = c_Y = c$ assume the same value (c).

Given the number of different techniques and their distinct computational approaches, a traditional manner to compare them accordingly to their community detection precision is through the use of random clustered networks [2, 11]. These networks are composed of N vertices divided into M groups (communities). The network is created following two probabilities, p_{in} and p_{out}. These probabilities are chosen to control the number of intra-community links (links between vertices belonging to the same community) z_{in} and the number of intercommunity links z_{out} (links connecting vertices of different communities) for a given network average degree $\langle k \rangle$. Adjusting these parameters, the proportion of intra-community links $z_{in}/\langle k \rangle$ and the proportion of intercommunity links $z_{out}/\langle k \rangle$ of the network are defined, where $(z_{in}/\langle k \rangle + z_{out}/\langle k \rangle) = 1$. In particular, many authors have employed networks with $N = 128$ vertices divided into $M = 4$ communities with the same size and $\langle k \rangle = 16$. In this way, starting from networks with $z_{out}/\langle k \rangle \approx 0$, where there are no intercommunity links or the connections are sparse, to $z_{out}/\langle k \rangle = 0.5$, where on average half of a vertex links are connected to vertices in the same community and the rest connected to vertices of other communities, it is possible to study and compare those algorithms. Figure 1 shows an example of four random clustered networks for different values of $z_{out}/\langle k \rangle$.

Figure 2 shows an illustration of the community detection process in a random clustered network composed of four communities. In Figure 2(b) it is possible to observe the synchronization phenomenon among neurons representing those communities. Once the synchronization is achieved, the communities can be easily identified accordingly to their distinct firing activities representing each group.

Figure 3 shows the proportion of nodes correctly classified as a function of the proportion of intercommunity links $z_{out}/\langle k \rangle$. This result was obtained from a set of 200 realizations on random clustered networks with $N = 128$, $M = 4$, and $\langle k \rangle = 16$, created according to the rule described in Section 1. Based on this results one can see that our model shows good results for a wide range of $z_{out}/\langle k \rangle$. When comparing to results from other techniques, such as the GN model proposed in [25], our oscillatory correlation model shows superior results, i.e. for a network with $z_{out}/\langle k \rangle = 0.4$, the GN model achieves a precision of 80% [2, 25] against 90% of our model. Results even superior are obtained when networks with $z_{out}/\langle k \rangle = 0.5$ are considered. In this case, the GN model presented a precision about 40% while ours showed $76 \pm 10\%$. Moreover, when put side by side the results presented in Figure 3 to those published in [2], we can check that the proposed model is among those which produces better results of community detection.

Next, two simulations performed on real networks are presented. Figure 4(a) shows the temporal series of all neuron representing their respective vertex of the network of friendship between individuals in the karate club studied in [24]. In this figure, one can see that after a number of cycles the communities can be identified by their own distinct time series. In order to facilitate the visual inspection of the time series produced here, in Figures 4(b)-(d), the time series for some interval of time are presented in a higher temporal resolution. Figure 4(b) shows the presence of two communities, except vertices number 9 and 10. This result is coherent to the one obtained in [1]. Figures 4(c) and (d) show two other

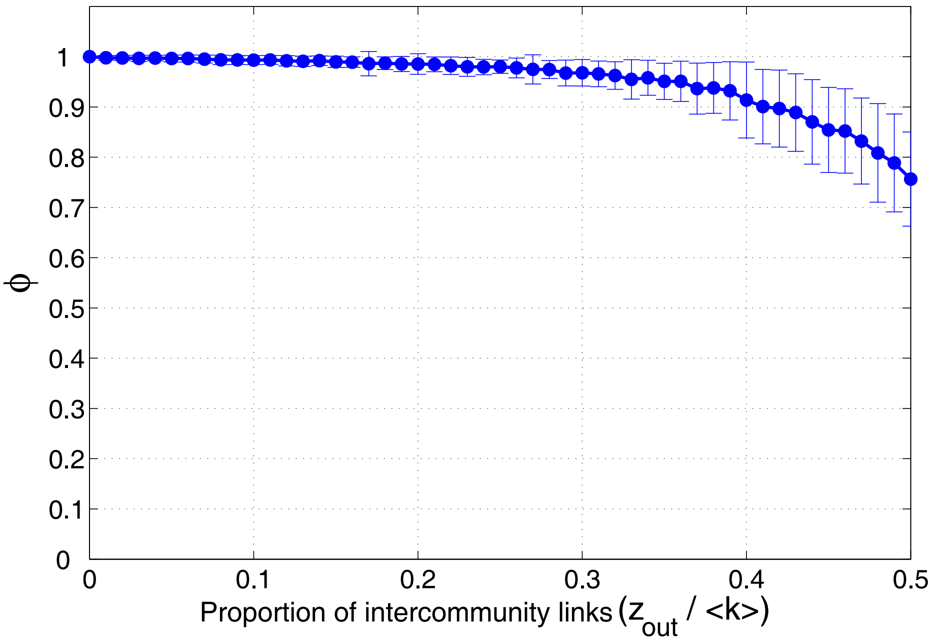


Figure 3 – Community detection rate (ϕ) using the oscillatory correlation technique applied to clustered random networks with $N = 128$, $M = 4$, and $\langle k \rangle = 16$. The x axis represents the proportion of intercommunity links ($z_{out}/\langle k \rangle$) and the y axis represents the proportion of nodes correctly classified. Each point in the trace is average by 200 realizations. The error bars represent the standard deviations

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instants of the simulation. Particularly in item (c), we can observe the presence of three communities, in which the vertices 5, 6, 7, 11, and 17 are grouped in a third community. Figure 5 shows a graphical illustration of this last division. It is worth noting that the same result was obtained in the study presented in [1, 25], which corroborates to show that our model can achieve coherent results.

The next simulation, presented in Figures 6 and 7, was performed on the dolphin social network [26]. This network was created base on the social ties observed in pairs of dolphins during several years of observation. Here, we followed the same methodology of the last simulation. First, Figure 6(a) shows the complete time series of this simulation and Figures 6(b)-(d) some periods with higher temporal resolution. By analyzing Figure 6(d) one can see the presence of three communities which matches to the results obtained in [1, 3]. Figure 7 shows this community detection result graphically.

The same network was also used in a simulation varying the inhibitory coupling strength c_Y while $c_E = 0.3$ was held constant. Figure 8 depicts the time series of this simulation. When $t = 0$, there is not inhibitory connections ($c_Y = 0.0$) and a global synchronization is observed. In $t = 2500$, c_Y is set to 0.2 and two communities are detected. Next, the inhibitory strength is increased to 0.4 and four communities are observed. Finally, $c_Y = 0.5$ and the network is further divided into 8 communities. This simulation provides some insights on how to perform hierarchical community detection using our model.

More simulations with synthetic networks have been conducted and results with similar quality have been obtained.

4. CONCLUSIONS

In this work, we have presented a new technique for community detection in complex networks based on the oscillatory correlation theory. The model has several interesting features. First, it is biologically inspired because it is developed by using the synchronization/desinchronization of coupled integrate and fire neurons. Second, it is efficient owning to the quick synchronization process. Finally, it can achieve high community detection precision. From this study we can conclude that the biologically plausible neural networks, which unify the dynamics of each individual neuron and the topological structure of neural interaction, has emerged as a powerful computational tool.

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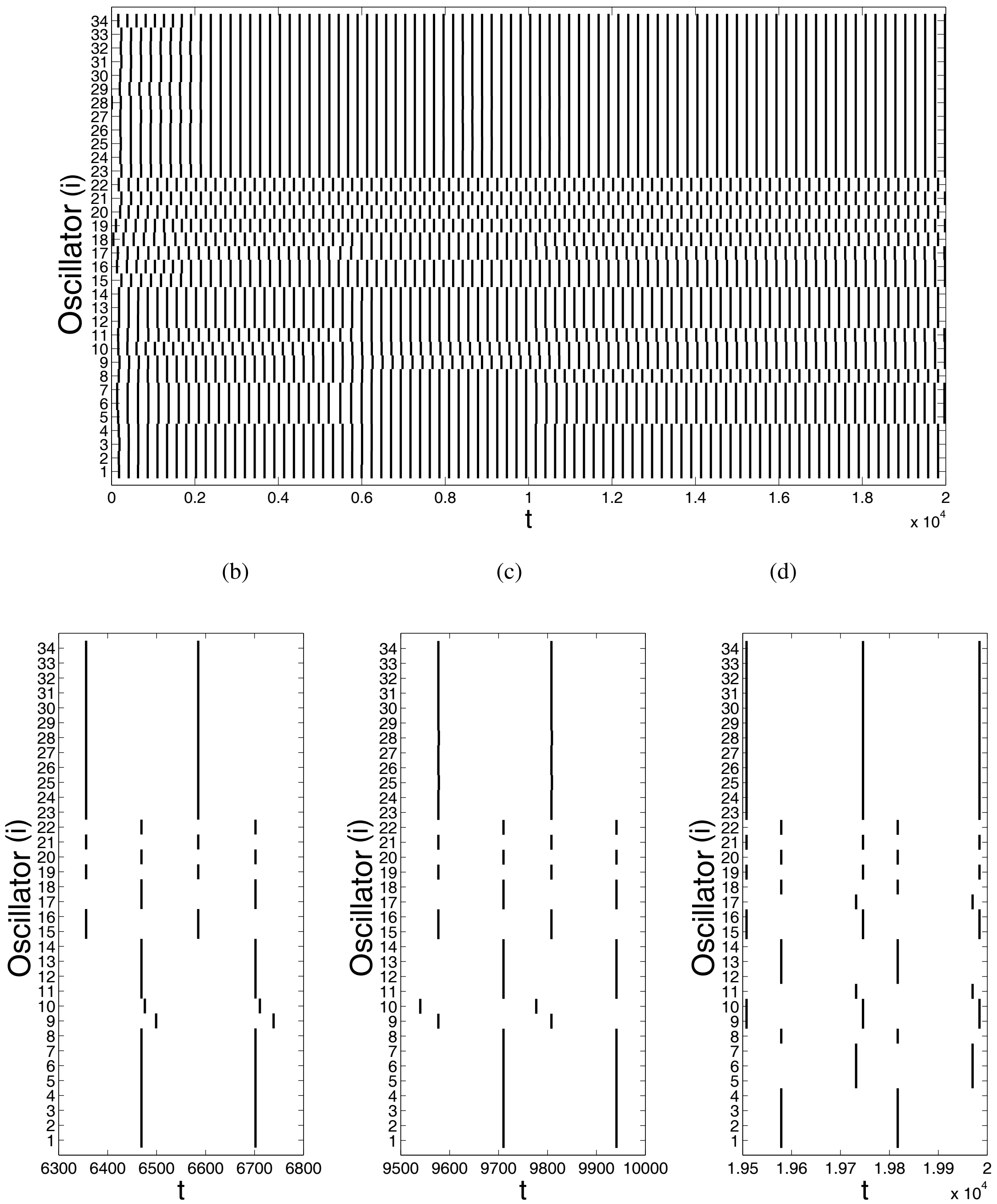

Figure 4 – Temporal series of oscillators representing vertices of the network of friendships between individuals in the karate club study [24]. $c = 0.1$. **(a) Complete temporal series. (b)-(d) Partial series in higher temporal resolution** t.

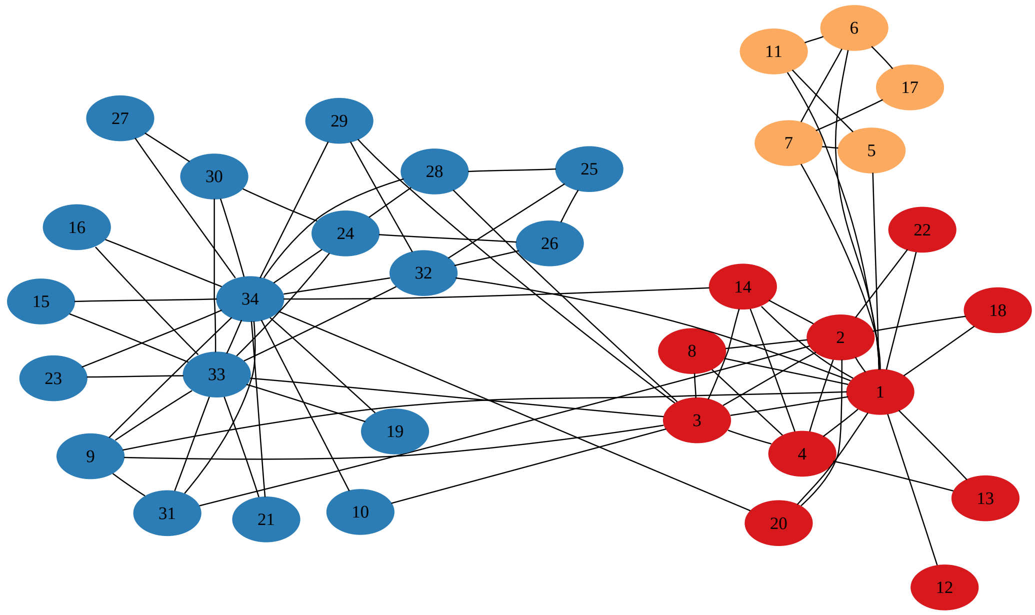

Figure 5 – Community detection result on the network of friendships between individuals in the karate club study [24]. $c = 0.1$.

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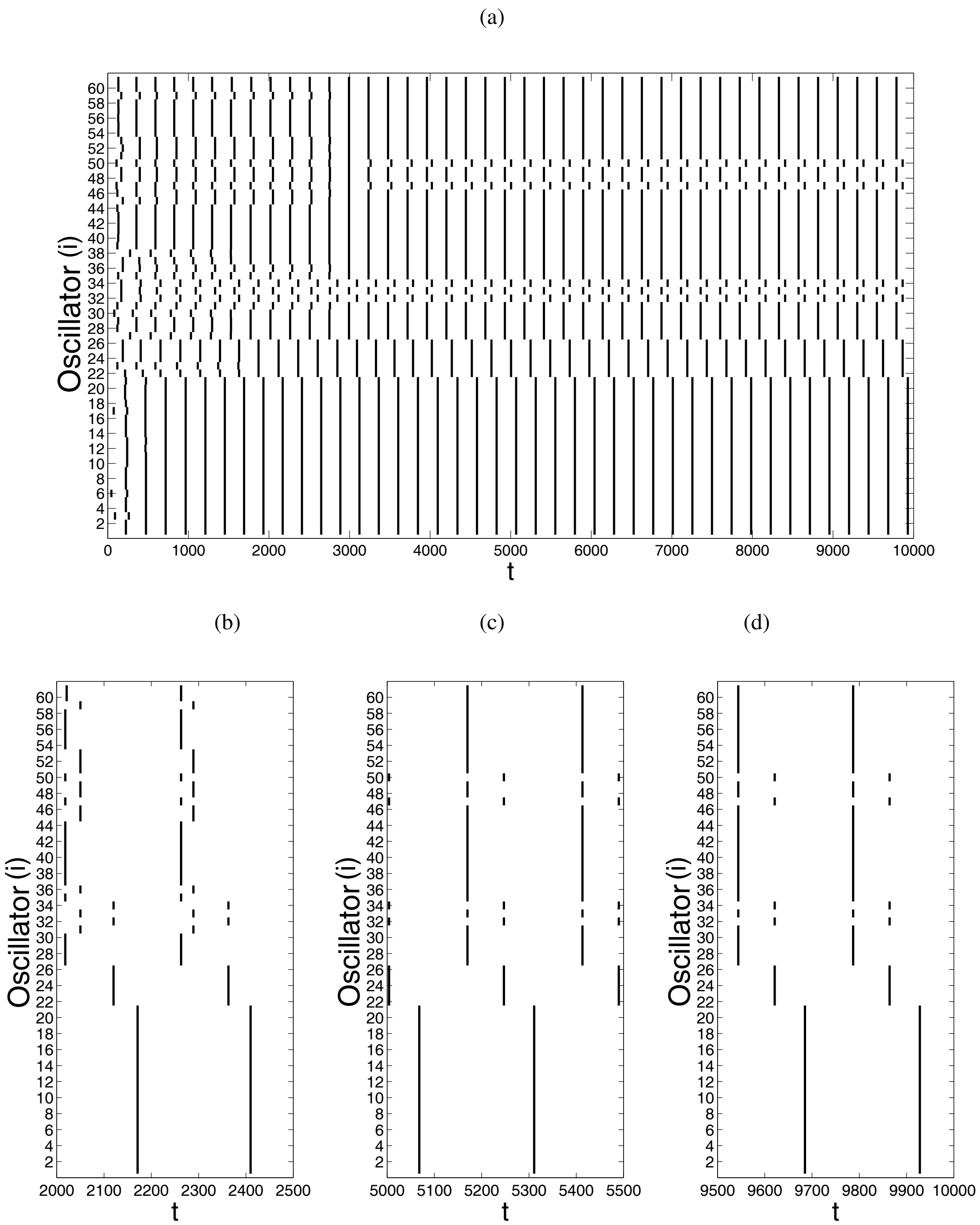

Figure 6 – Temporal series of oscillators representing vertices the dolphin social network [26]. $c = 0.3$. **(a) Complete temporal series. (b)-(d) Partial series in higher temporal resolution** t.

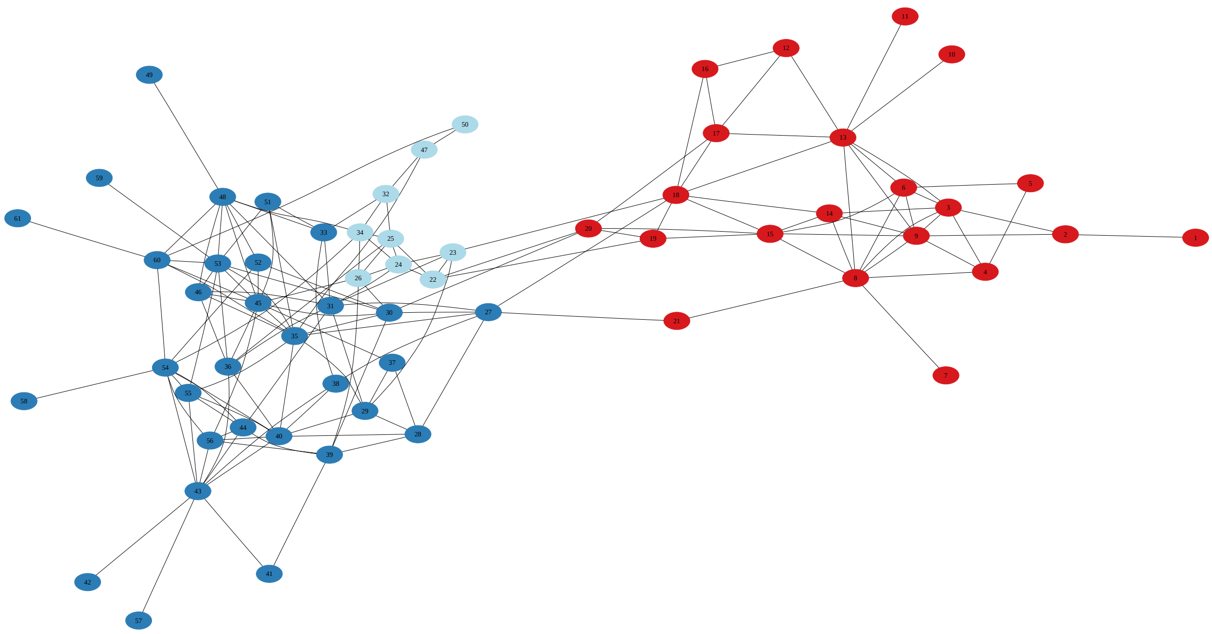

Figure 7 – Community detection result on the dolphin social network [26]. $c = 0.3$**.**

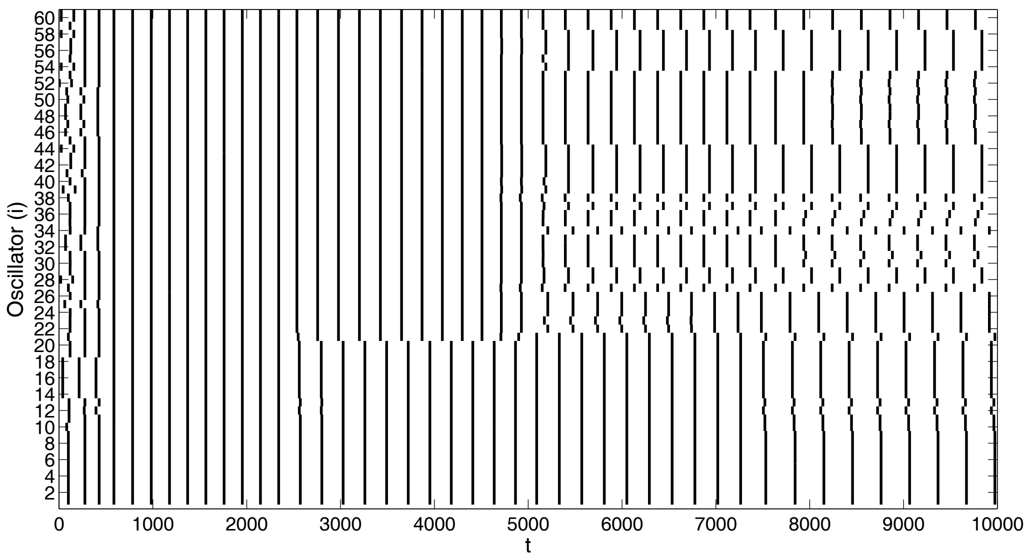

Figure 8 – Temporal series of oscillators representing vertices the dolphin social network [26]. c_E **is hold constant at** 0.3**;** c_Y **assumes different values during the simulation: from** $t = 0$ **to** $t = 2500$**,** $c_Y = 0.0$**;** $2500 < t \leq 5000$**,** $c_Y = 0.2$**;** $5000 < t \leq 7500$**,** $c_Y = 0.4$**;** $t \leq 5000$**,** $c_Y = 0.5$**.**

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