
# Synchronization Analysis in Models of Coupled Oscillators

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# Summary

# Introduction

- Visual Attention is a technique used by biological neural network systems developed to reduce the large amount of visual information that it is received by natural sensors [5].

- In 1981, von der Malsburg [13] suggested that each object is represented by the temporal correlation of neural firing activities, which can be described by dynamic models

# Introduction

- A natural way of representing the coding of the temporal correlation is to use synchronization between oscillators.

- Objective: Study of synchronization in some biological neurons' models which exhibit chaotic behaviors, by using a coupling force  between the oscillators as in Breve et. all work [3]

- The motivation is to use this sync method for visual selection of objects that represents sync neurons' models, while the rest of the image is unsynced.

# Phase Synchronization

- The phase synchronization of two oscillators *p* and *q* happens when their phases difference $|\varphi_p - \varphi_q|$ is kept below a certain phase threshold C.
- So as t → ∞, $|\varphi_p - \varphi_q| < C$. The phase *i* at time $t_i$ is calculated as following [11]:

$$\varphi_i = 2\pi k + \frac{t_i - t_k}{t_{k+1} - t_k} \qquad (1)$$

- where k is the number of neural activities prior to time $t_i$, and $t_k$ and $t_{k+1}$ are the last and the next times of neural activity, respectively.

# Phase Synchronization

- So that two oscillators can synchronize with each other, a coupling term is added to the dynamical system as the following:

$$\begin{aligned} \dot{x}_j^p &= F_j(\boldsymbol{X}, \boldsymbol{\mu}) + k\Delta_{p,q} \\ \dot{x}_j^q &= F_j(\boldsymbol{X}, \boldsymbol{\mu}) + k\Delta_{q,p} \end{aligned} \quad (2)$$

- Where $\dot{x}_j^p$ and $\dot{x}_j^q$ are the time evolution of the $x_j$ state of the *p* and *q* oscillators. $F_j(\boldsymbol{X}, \boldsymbol{\mu})$ is the behaviour's rate and $k\Delta_{p,q}$ is the coupling term, where k is a coupling force and $\Delta_{p,q}$ is the difference between the states:

$$\Delta_{p,q} = x_j^q - x_j^p \quad (3)$$

# Methodology and Models

- The proposed models for the attention system are a two-dimensional network of neural models' dynamical systems with coupled terms.

- Dynamical Systems: Hodgkin-Huxley [8], Hindmarsh-Rose [7], Integrate-and-Fire [10], Spike-Response-Model [6]. It was used the 4th Order Runge-Kutta numerical method.

- Discrete Models: Aihara’s [1], Rulkov’s [12], Izhikevic [9] and Courbage-Nekorkin-Vdovin [10].

- Search for chaos by varying the parameters values in $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_i, \ldots, \mu_N)$ or adding a white noise at the models.

# Methodology and Models

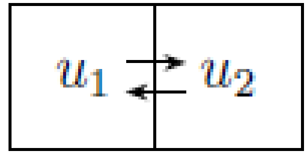



Fig. 1: Two Oscillator Problem

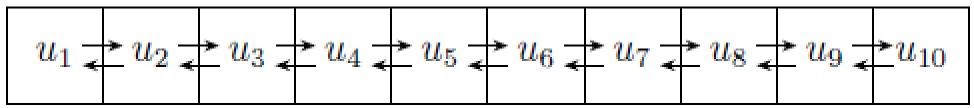



Fig. 2: Vector of Oscillators Coupled

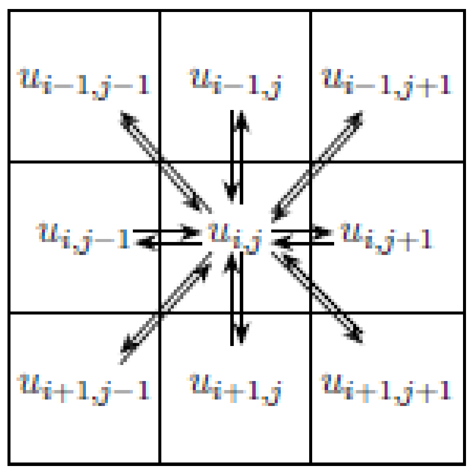



Fig. 3: Grid of Oscillators Coupled

# Methodology and Models

Coupling Force Variation: some oscillators were strongly coupled and others weakly, so that the first were synchronized and hence clusterized.

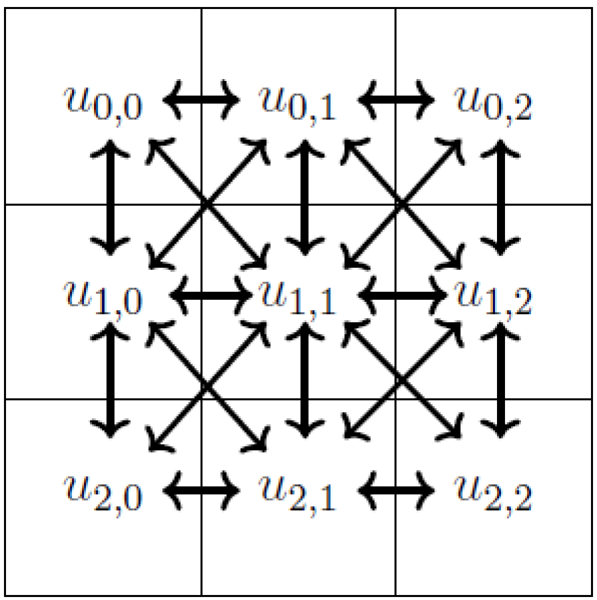



Fig. 4: Grid of Neurons

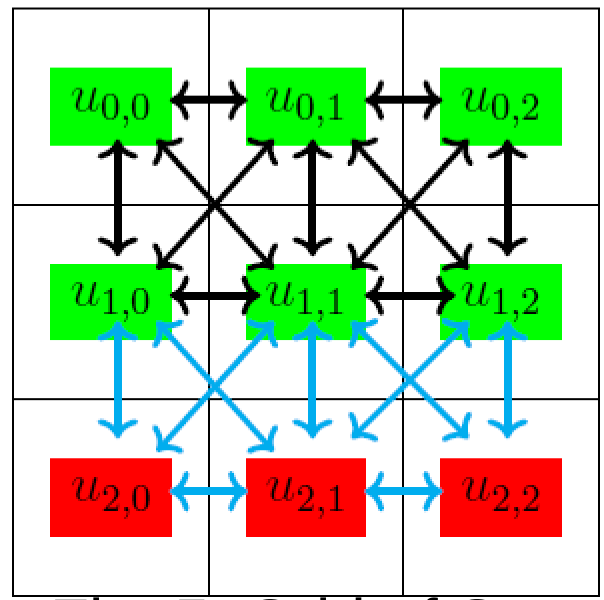



Fig. 5: Grid of Sync Neurons and Unsync.

# Results

Chaotic and stochastic trajectories to represent different neurons and pixels.

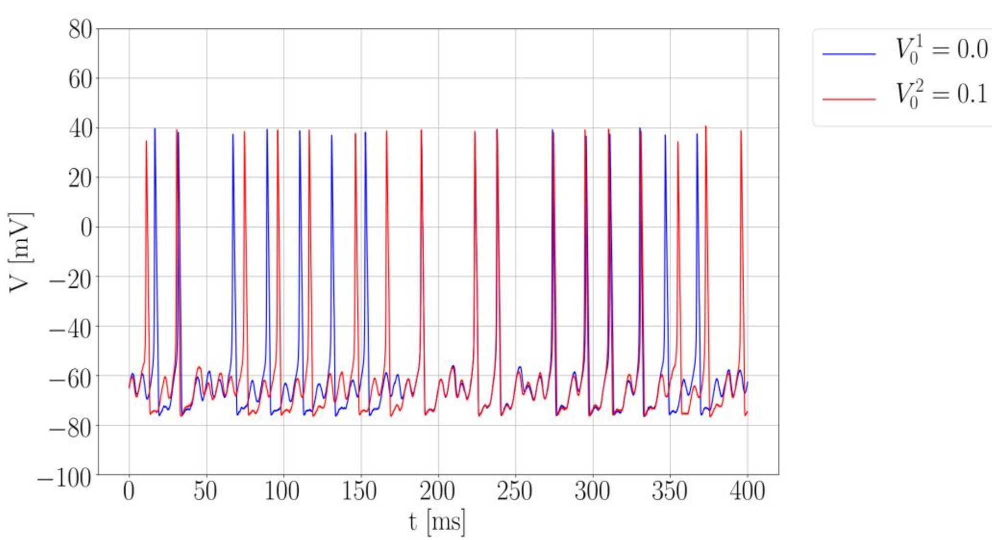



Fig. 6: Stochastic Hodgkin-Huxley

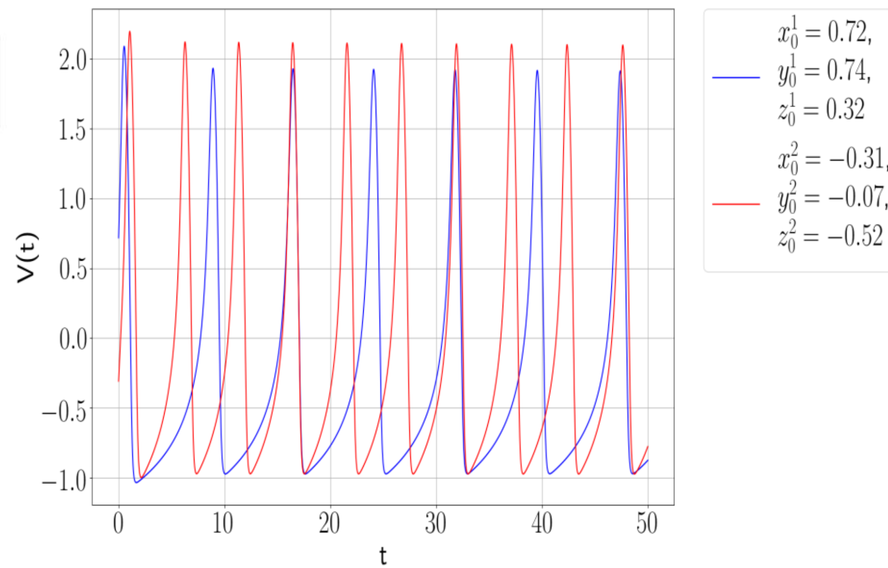



Fig. 7: Chaotic Hindmarsh-Rose

# Results

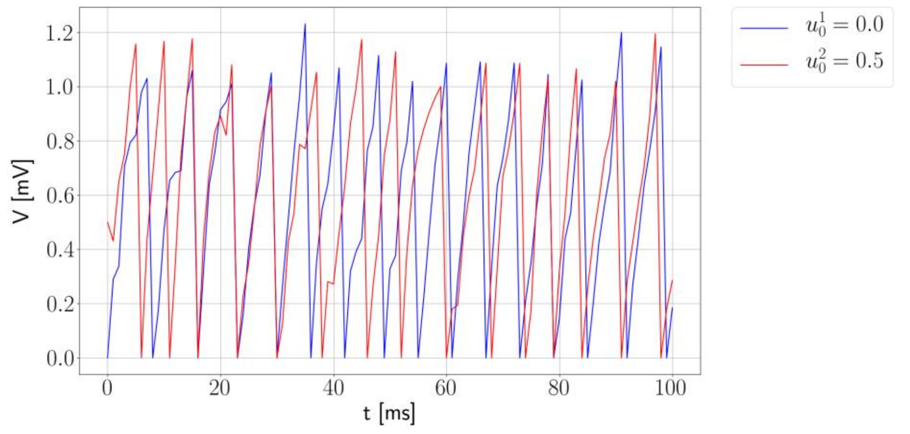



Fig. 7: Stochastic Integrate-and-Fire

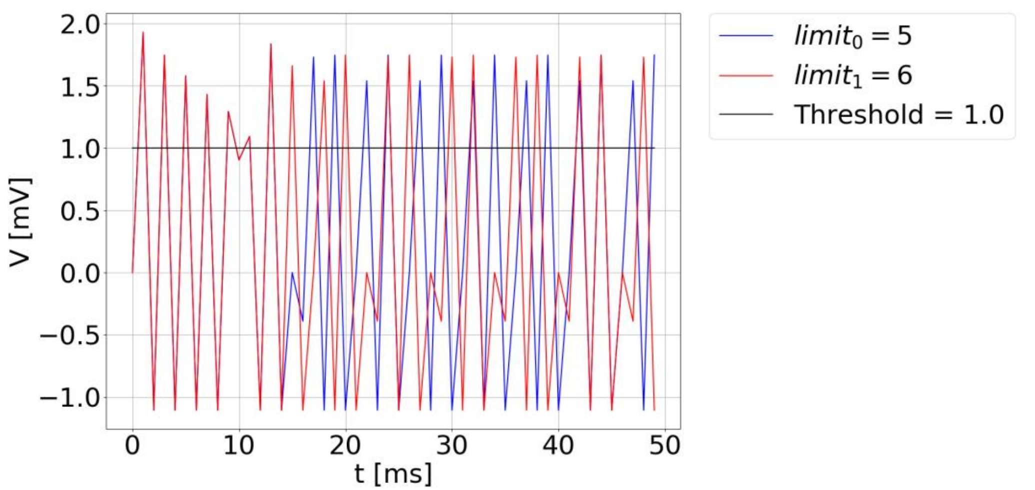



Fig. 8: SRM with different limit times

# Results

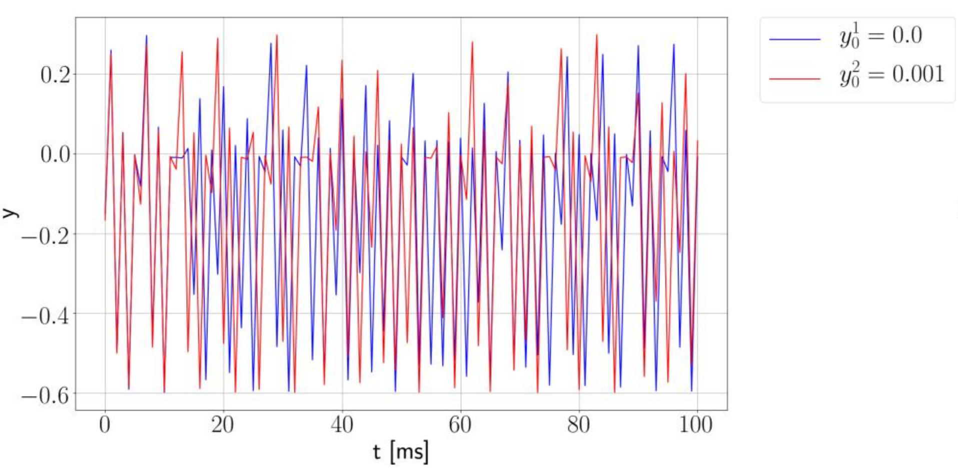



Fig. 9: Chaotic Aihara

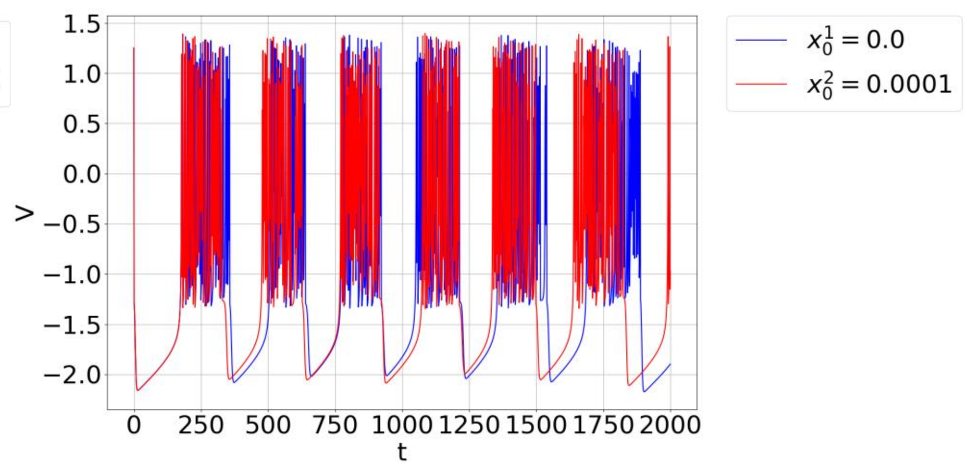



Fig. 10: Chaotic Rulkov

# Results

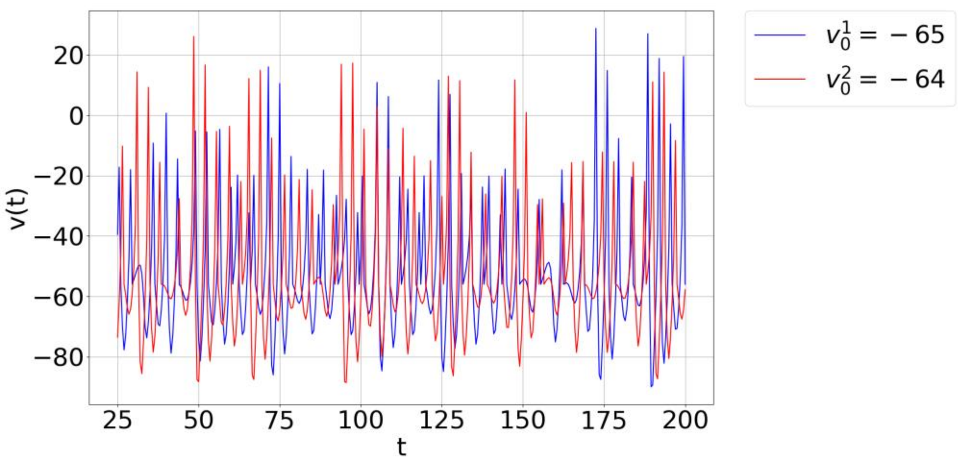



Fig. 11: Chaotic Izhikevic

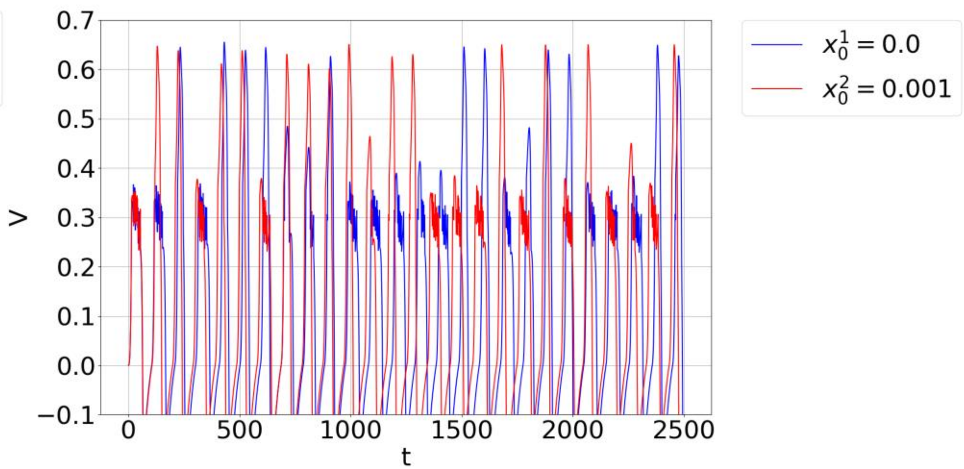
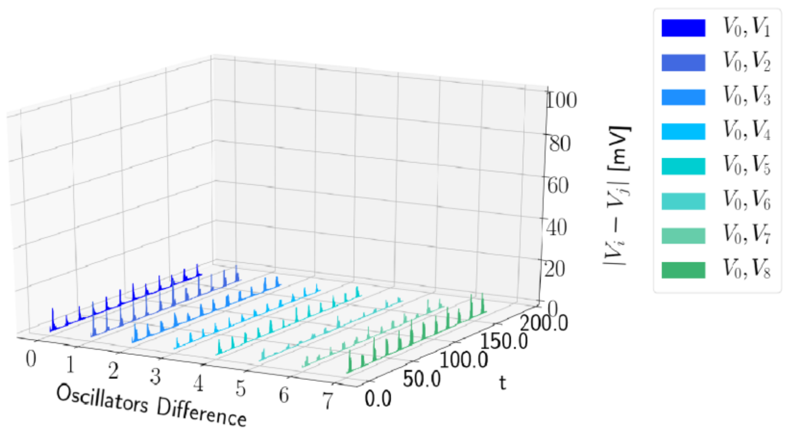


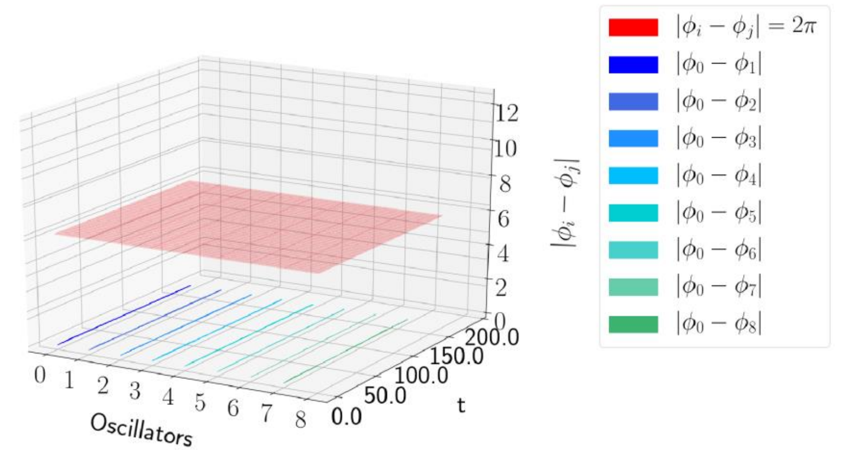

Fig. 12: Chaotic CNV

# Results

Trajectories and phases difference of a grid of oscillators with phase threshold at $2\pi$ [2].



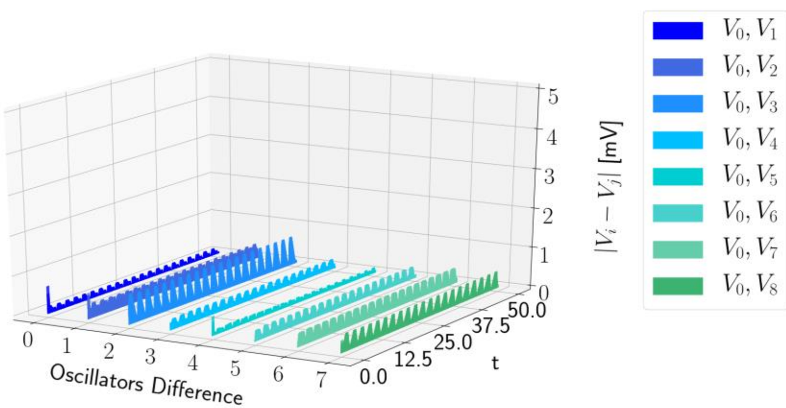

(a) Trajectories Difference



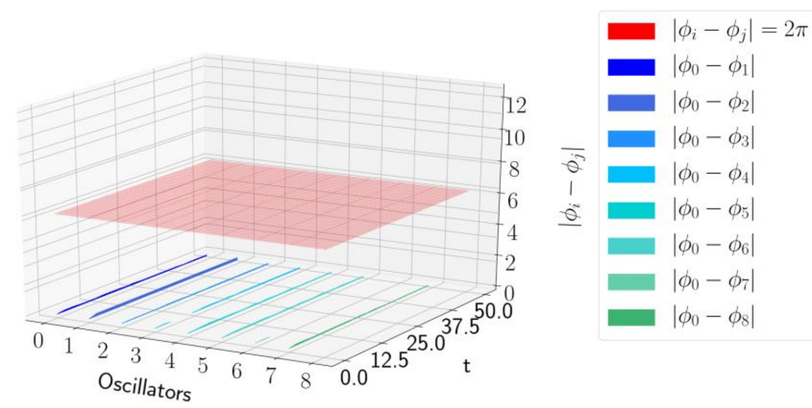

(b) Phases Difference

Fig. 13: Hodgkin-Huxley Model

# Results



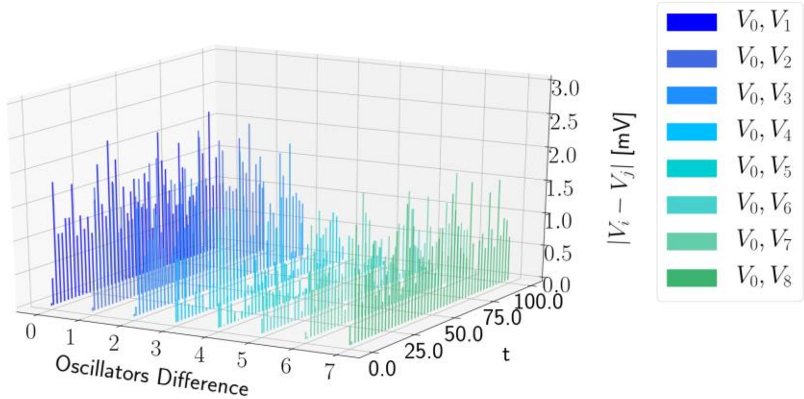

(a) Trajectories Difference



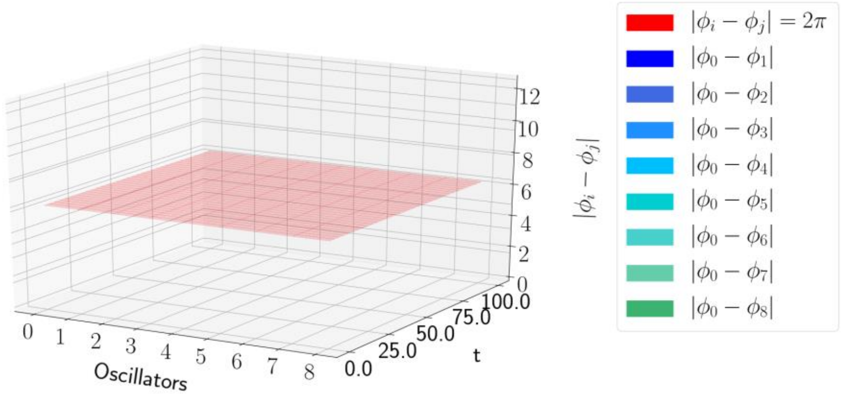

(b) Phases Difference

Fig. 14: Hindmarsh-Rose Model

# Results



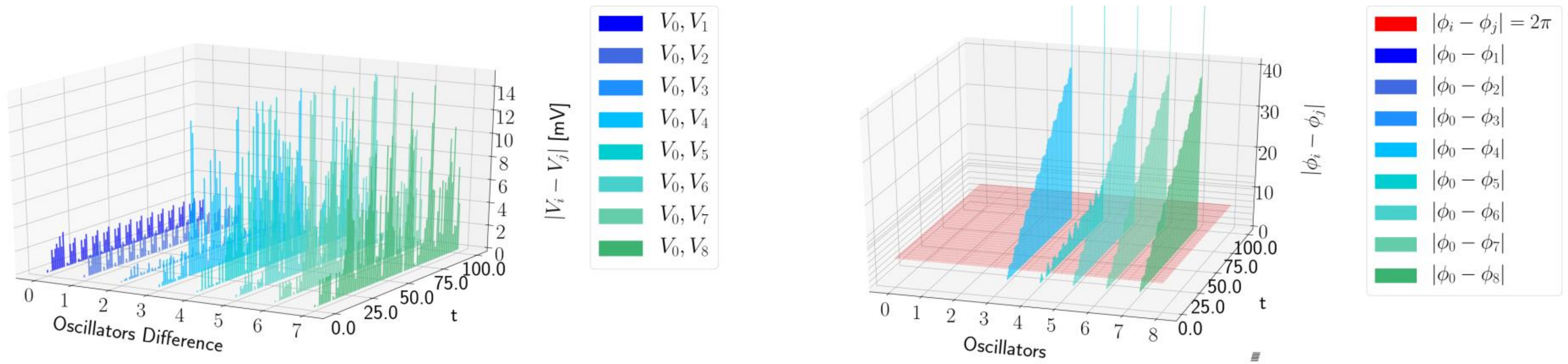

(a) Trajectories Difference




(b) Phases Difference

Fig. 15: Integrate-and-Fire Model

# Results

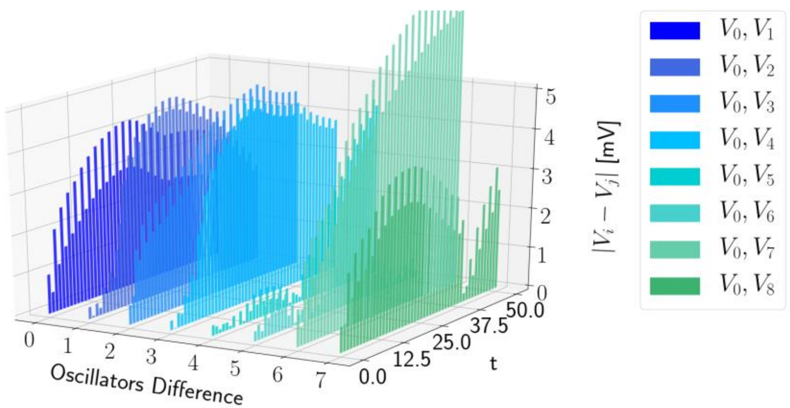



(a) Trajectories Difference

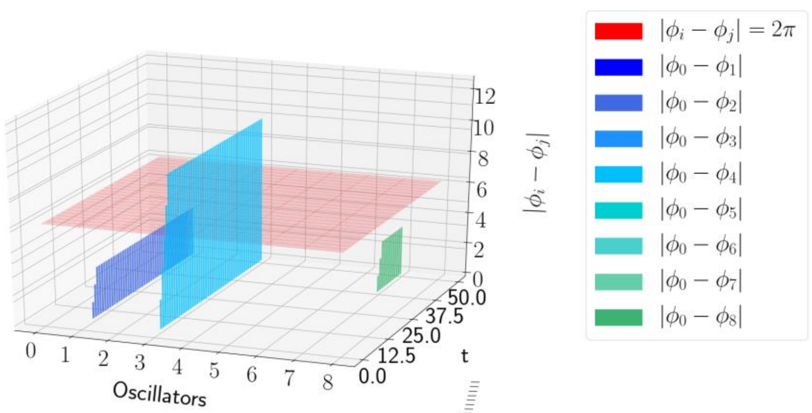
(b) Phases Difference

Fig. 16: Spike-Response-Model

# Results



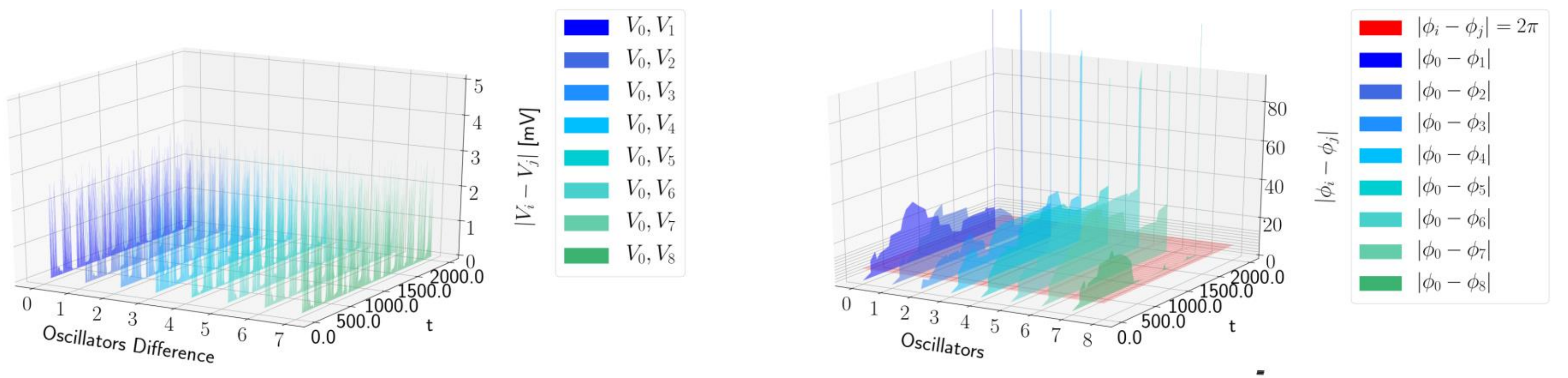

(a) Trajectories Difference




(b) Phases Difference

Fig. 17:Aihara’s Model

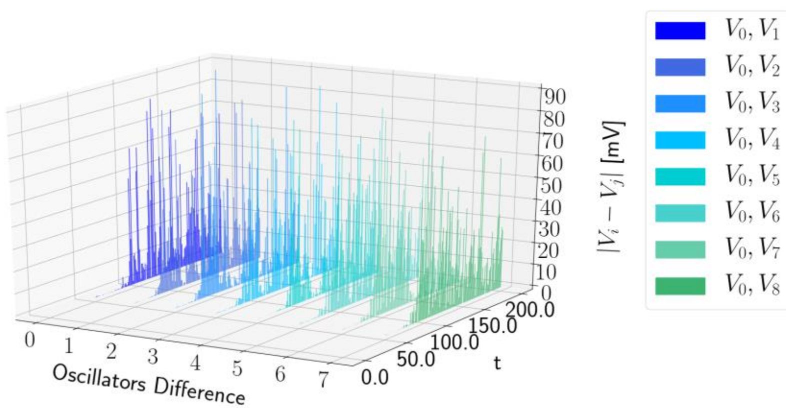
# Results



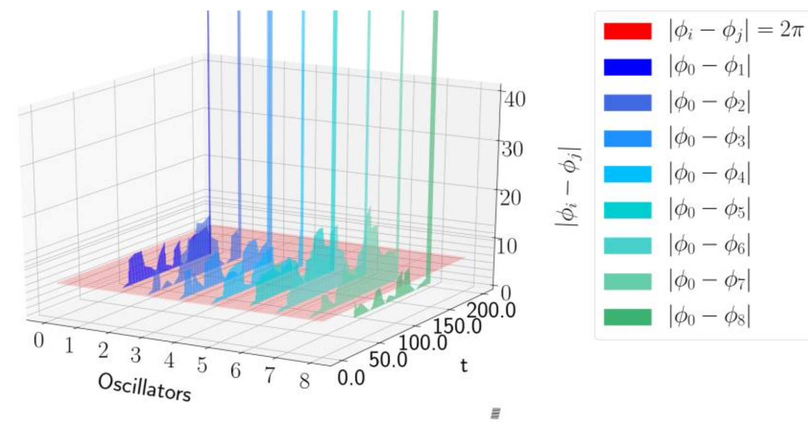

(a) Trajectories Difference (b) Phases Difference

Fig. 18: Rulkov’s Model

# Results



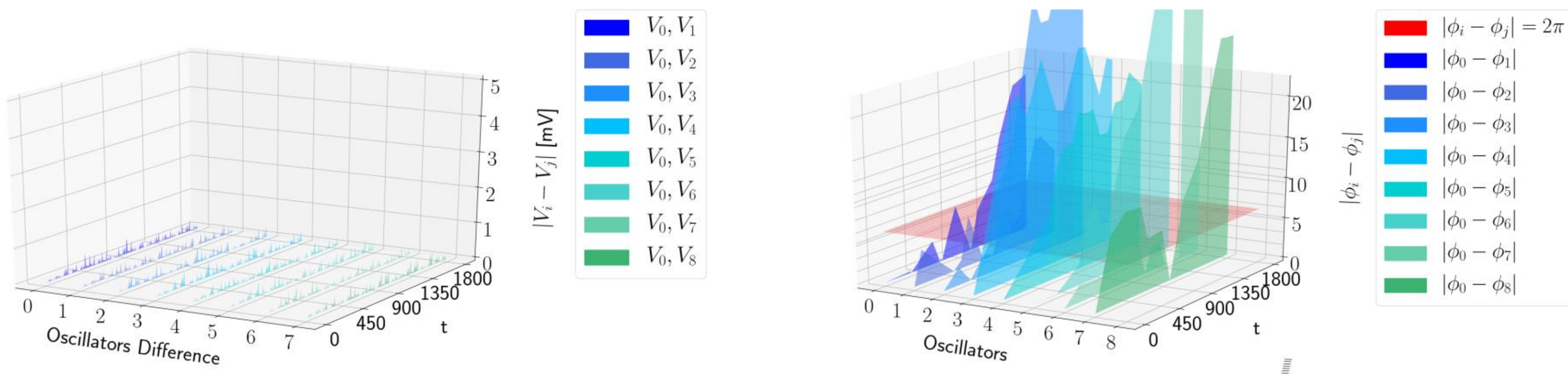

(a) Trajectories Difference




(b) Phases Difference

Fig. 19: Izhikevic’s Model

# Results



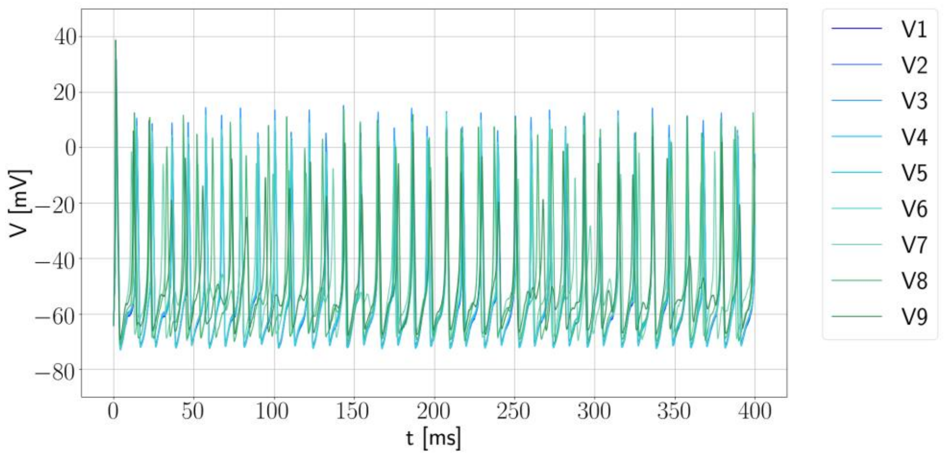

(a) Trajectories Difference

(b) Phases Difference

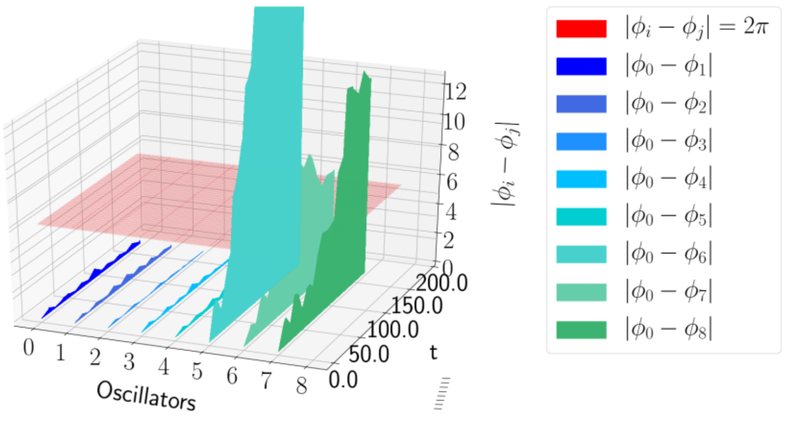
Fig. 20: Courbage-Nekorkin-Vdovin Model

# Results

Trajectories and the phases difference of the models (Hodgkin-Huxley, Hindmarsh-Rose and Integrate-and-Fire) in a grid with sync and unsync oscillators.



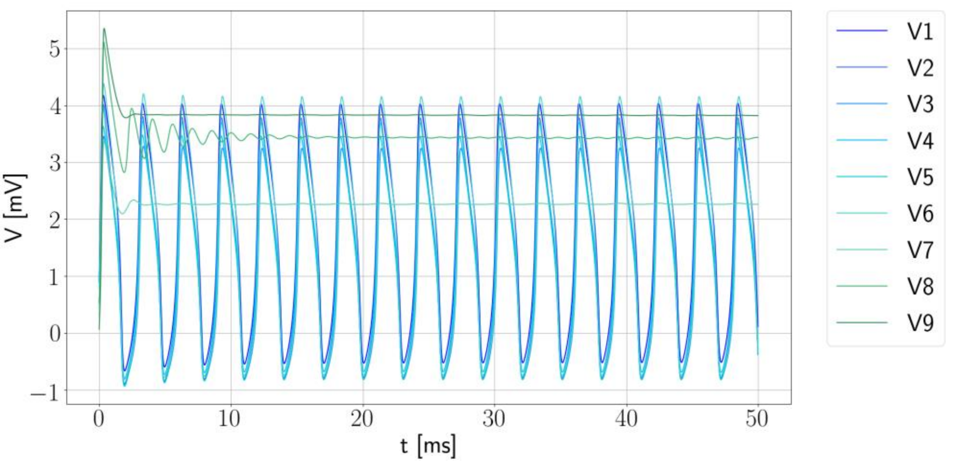

(a) Synchronized and desynchronized Trajectories



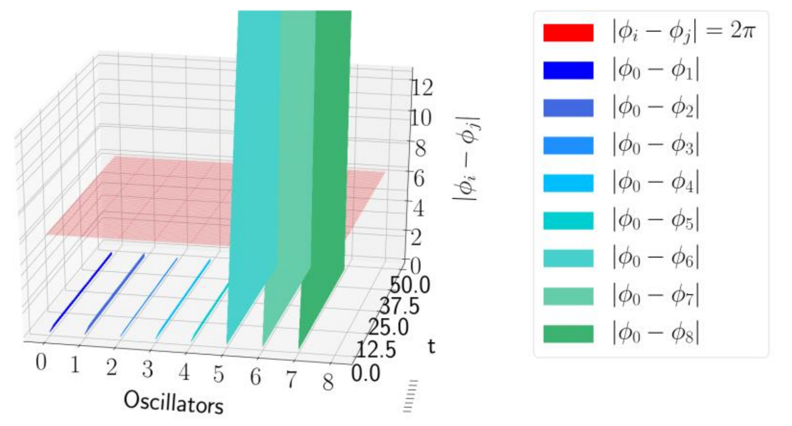

(b) Phases Difference

Fig. 21: Hodgkin-Huxley Model

# Results



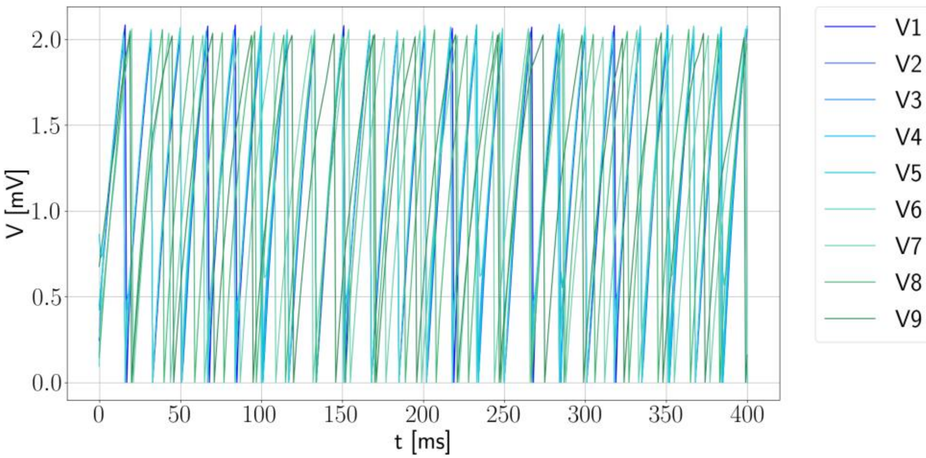

(a) Synchronized and Desynchronized Trajectories



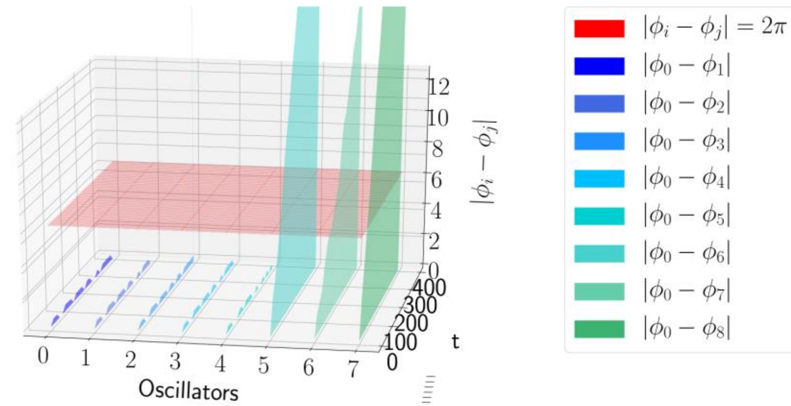

(b) Phases Difference

Fig. 22: Hindmarsh-Rose Model

# Results




(a) Synchronized and Desynchronized Trajectories




(b) Phases Difference

Fig. 23: Integrate-and-Fire Model

# Conclusions

- Discrete time models didn’t synchronizes. Continuous time models synchronizes.
- Spike-Response-Model synchronizes without a coupling force, only considering the arrival time of presynaptic stimuli. But did not show chaos behavior.
- The continuous models tested for the synchronization and desynchronization for a cluster formation depending on the coupling force showed a potential solution for a visual selection mechanism for an attention system.

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